Kevin Mauser

Shane Randa

Zane Ranney

Unit Plan Rationale: Quadratic Models

The concept of a quadratic is a difficult one for many students to grasp. All their life, students have been exposed to plotting points that create straight lines and looking at linear functions. The truth is quadratics are all around us, from the projectile of throwing a baseball to maximizing the area of fenced in yard. With the students in our class, we need to get them motivated to learn. They show endless potential, it is our job as educators to get them engaged in the classroom and for them to realize that math is essential and can even be fun.

That being said, we want to get them to see quadratics in real-life from day one. We plan to use the motion sensor exercise that we were exposed to during our technology day to get the students up and moving around in the classroom, actually “being” a quadratic equation. Our goal here is to make math fun. We fully expect the students to have prior knowledge of functions; that if I plug in a value for x, what is my y value going to be? The idea of inputs and outputs is crucial when getting into quadratics, and eventually cubic functions. There is always improvement when it comes to mathematical reading, especially with the few students in our class that struggle in this area. Our goal is improve in this area by exposing the students to word problems that they can relate to, at first helping them break them down, and then requiring them to do so individually. We have added a section to this chapter, maximization, because this topic is one that can greatly improve mathematical reading due to all of the real life examples it has.

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| **Date** | **Brief Description of Content and Lesson** | **?Technology, Special Activities, Manipulatives, Problem-Based, Instructional Strategies?** |
| May 5th | We will be doing an activity involving motion sensors, both to record height off the ground and horizontal distance away from the sensor. First, students will jump off the ground, letting the sensor record their height over time. Then, placing the motion sensor at the higher end of a slanted table (placing the legs of one side on books for example), students will roll the object at the sensor, which will record the distance as it gets closer and then farther away. Both of these experiments should produce parabolas (one pointing up, one pointing down), and students will be able to see the real world rationale for analyzing graphs such as these. We can even discuss other instances (falling, throwing a ball, etc.) where these graphs make sense. | We will need enough motion sensors so each group of 2 or 3 can have their own. We are also anticipating that their desks will have 4 legs, so we can prop up 2 of them. Finally, we will need to make sure that students have their own graphing calculators to work with the sensors (whether their own or we just bring in a class set). |
| May 6th | Today will be our first lesson, and it is an introduction to quadratic equations. We will introduce the idea of the zero product property, and from there we will relate a quadratic function to the product of two linear functions. For example, how the x-intercepts are the same. This lesson will be discussion based with a brief activity. | * An ELMO * Graphing calculators for each student (their own or from a class set) * Strips of paper or rulers, to cover up different parts of the graph (to emphasize certain areas) * The 2 activity worksheets from the main lesson plan |
| May 7th | The lesson for the day is a both a problem-based and a procedural one. This will deal with the idea of finding “roots” in a function when the graph is already given. This will be done both by visually analyzing the roots and learning how to find roots via a graphing calculator. We will also some benefits to finding roots (such as solving any quadratic equation by placing all the terms on one side and treating that as a function). The “factored” form of a function will also be introduced. | * Graphing calculators for each student (their own or from a class set) * An ELMO to show graphs, and a projection screen that can display a graphing calculator screen (projector or SMART board) |
| May 8th | Today’s lesson builds off of the previous day and will be about converting quadratic functions into “factored” form algebraically. We will discuss removing common factors, and go over the process of factoring trinomials into 2 separate monomials. As a class, we will learn different techniques, and discuss the benefits to using these techniques (like how finding intercepts is much easier to do in “factored” form). | * “Warm-up” worksheet for each student as they enter class. * ELMO for going over the warmup. * Either a SMART board, chalkboard, or overhead (just some area of focus) |
| May 9th  (half-day) | Since today is a half-day, we will not introduce any new material. Instead, students will take a small 15-20 minute quiz on the material covered in the few lessons prior (so quadratic equations, finding roots, factoring, and comparing general and factored form). | * Graphing calculators for each student (their own or from a class set) * Quizzes for each student. |
| May 12th | We will begin the day by going over the quiz from Friday, and clearing up any misconceptions observed from the assessment. Afterwards we will begin a new lesson on translating quadratic functions and introduce the vertex form, and its purpose (to show where the vertex is very clearly). The lesson will conclude with us demonstrating how to convert quadratic functions from vertex form to general form, and vice versa. | * Graphing calculators for each student (their own or from a class set) * Either a SMART board, chalkboard, or overhead (just some area of focus) * Assessment day: going over quiz on last week’s material |
| May 13th | Today is our technology day, where students will be in the lab exploring different ways to manipulate a quadratic function (in multiple forms), and the results that follow. By completing the follow-along worksheet, students will see parabolas come to life through animation, and discover how the functions are adjusted when certain changes are made. | * Computer lab or set of laptops with Geogebra program. * Follow-along worksheet for technology activity |
| May 14th | Today, we will introduce students to the quadratic formula, used to find the roots of any quadratic function. The fact that this formula can find the roots of any function, with or without a calculator, even those we cannot factor, gives us an opportunity to make this a problem-based lesson. There will be no special activities today. | * Graphing calculators for each student (their own or from a class set) * Either a SMART board, chalkboard, or overhead (just some area of focus) |
| May 15th | Today we conclude our lesson involving the quadratic formula by going a bit more in depth. We will go further than just defining the formula and its purpose, and will instead focus more on different scenarios of the formula for different functions, mainly discussing the discriminant. Mainly, we will discuss the scenarios when the discriminant is 0, or when it is negative (introduce the idea of imaginary numbers). Following this, we will briefly wrap up the lessons for the week and hand out the review sheet to be completed for homework. | * Graphing calculators for each student (their own or from a class set) * Either a SMART board, chalkboard, or overhead (just some area of focus) * Review sheet for weekly quizzes |
| May 16th | Today is the second and final weekly quiz of the unit. After the 15-20 minute quiz, we will begin our problem-based lesson on maximization. This will involve real-world scenarios (ball being tossed, maximum area for given fence perimeter, etc.) and sort of “bring it all around” and give the previous 2 weeks even more relevance. | * Graphing calculators for each student (their own or from a class set) * Assessment: weekly quiz * Either a SMART board, chalkboard, or overhead (just some area of focus) |
| May 20th | Today we are finishing up the lesson on maximization. We will first conclude the area problem posed last Friday by having students work on it in groups, and then as a class. We will then pose another problem on maximization (motion), and solve this in a similar way. We conclude the lesson with a discussion on minimization (cost) and hand out the corresponding worksheet to be started in class and finished for homework. | * Graphing calculators for each student (their own or from a class set) * Assessment: minimization worksheet * Either a SMART board, chalkboard, or overhead (just some area of focus) |
| May 21st | This will be the final lesson that we will cover in the unit. It will be on cubic functions. We will relate cubic functions to quadratic ones both graphically and algebraically, and will discuss transformations to cubic functions. Lastly, we will discuss finding roots of cubic functions and how the cubic factored form looks algebraically. | -SMART Board  -Graphing calculator for each student (either their own or one from the class set)  -Mini dry erase boards (one for each student)  -Manipulative: 4-unit cube for demonstrating volume. |
| May 22nd | Today we are playing a Jeopardy style review game, to prepare for the unit test. We will have categories representing different ideas from the unit (factoring, maximization, quadratic formula, etc.), and each category will have several different questions (valued at different points) that relate to a given idea. This will thus cover all the ideas from the unit, with assessment questions similar to those that will be on the test. | - About 31 questions (5 questions for 6 categories, and 1 more difficult and all-encompassing questions for Final Jeopardy  - Note cards to be placed on the board to represent a real Jeopardy board.  -Visual Aide (projector, SMART board, or chalkboard, etc.) |
| May 23rd | Today is test day! This is the day that we have the opportunity to assess our students’ learning with a full period test encompassing all the main ideas of the unit (solving quadratic equations, analyzing quadratic functions, cubic functions, etc.). There will be two separate parts of the test. One will allow the use of graphing calculators (maximization, using tables) and the other will not (factoring, analyzing different forms) | -Students only need pencils, and we will provide scratch paper to allow more space for students to do work.  -Graphing calculator for each student (they can check out one from us if they don’t have one). |

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| Day #: 1 | Lesson Title: Motion Sensor Activity \*technology activity\* |
| Goal: | Engage students with an activity that will introduce them to the graphs of quadratic functions using real data that the students create. |
| Objectives: | Students will get in groups and take turns using the motions sensors to graph their own data.  Students will predict their graphs before taking data and compare and contrast graphs of others.  Students will be able to describe and interpret what their graph means, as well as reproduce them. |
| Lesson Summary (one paragraph maximum) | Students will get into groups and use motion sensors. They have used these for linear equations, so they are familiar with how to use them. Students will be directed to point the motion sensor at the ground, jump off of the ground and record their data by sketching the graphs. They will then interpret their graphs and record what the graphs mean in terms of distance over time, as well as why their graphs are shaped like a parabola. |
| HW | No homework |

## Name: Zane Ranney

**Class/Subject:** Algebra I

**Date:**  May 6th, 2010

**Student Objectives/Student Outcomes:**

* Explain the relationship between linear factors of a quadratic functions and the graph of the functions
* Based on the graph of two lines, sketch the parabola that is the product of two linear expressions
* Given the graph of a quadratic function, find the equations of the lines that could be components of the polynomial

**Content Standards:**

**8.A.3b** Solve problems using linear expres­sions, equations and inequalities.

**8.B.3** Use graphing technology and algebraic methods to analyze and predict linear relationships and make generalizations from linear patterns.

**Materials/Resources/Technology:**

Strips of paper or rulers

Graphing calculators

Building Quadratic Functions Activity Sheet

Working Backwards Activity Sheet

\*Make sure to manually draw the missing linear graphs as needed before handing out worksheets.

**Prior Knowledge:**

Students should be familiar with linear functions and graphing linear functions. Students should know slope intercept form of linear equations and be familiar with x and y intercepts. Students should also know how to multiply binomials and solve an equation for x.

**Teacher’s Goals**:

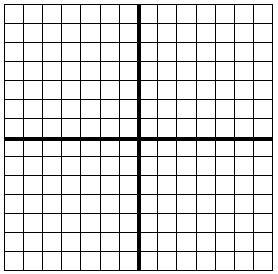
Enable students to understand the relationship between linear and quadratic functions. Students should be able to graphically construct a quadratic equation from two linear equations and vice versa. Each student should be working in pairs and filling out his or her worksheet as the lesson is taking place.

**Time**

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| 5 minutes. | **Start of Class:** At the beginning of class, prompt a discussion about yesterday’s activity involving motion sensors (\*hook was motion sensor activity). Make sure students are participating and at least saying if they liked/disliked the activity and why. If a student makes a valuable comment that students should hear, revoice the comment to the class. Assess students understanding of the purpose of the motion sensor activity. Connect yesterday’s class to the motion sensor activity that was used to introduced linear equations earlier in the year, and ask the class how they are related. Conclude that yesterday’s activity was an informal way of introducing them to a new type of function they will be exploring for this unit. Explain that today they will be exploring this new type of function in more detail and building on their knowledge of linear functions. |
| 5 - 7 min. | **Introduction of Lesson:** Informally ask them what xy = 0 means and write it on the board. If two quantities multiply to get zero, does x = 0 or y = 0, or both? Move on to (x-2)(x-3) = 0. When these two quantities multiply to get zero, does x-2 = 0 or x-3 = 0, or both? Mention the Zero Product Property if they are having a hard time agreeing on an answer. After some discussion, explain that either one of the quantities must be 0, or both must be 0 by the Zero Product Property. If need be, do an example without variables (ex: (2)(3) = 0? (2)(0) = (0)(3) = (0)(0) = 0).  Afterwards, break the quantities into two linear equations and have the class solve for x. Briefly check their solutions and emphasize that these are linear equations and that they should know how to solve for x, as well as graph them on the xy plane. Discuss if this function is linear? If not, why not?  Put the students in pairs and hand out the “Building Quadratic Functions” worksheet and explain that (x-2(x-3) = y is actually a quadratic function. Explain that quadratic functions are obtained by multiplying two linear expressions together. |
| 30 - 35 min. | **Lesson Instruction:** Voice expectations for student behavior while working in pairs. Use the Elmo to walk through the worksheet with the students. Make sure the students that struggle with reading are focusing on you for visual examples as well as filling out their worksheets. Have students start by identifying the linear function on the worksheet and putting it in y = mx+b form. Have them factor out the slope so that it is in slope/x-intercept form y = m(x+b/m). This helps students focus on the x intercept of the graph. Walk around and see that everyone has the correct equation in this form. Have them choose another function and have them write it in slope/x-intercept form, and graph it on the same graph as the original line. Make sure to tell students not to make their line so extreme (have an x and y intercept shown on the graph). Invite a student to the board to draw their function on the Elmo worksheet. Throughout the worksheet, address the graph as “[Student’s name]’s graph”  Have the students then make predictions of how a new function formed by product of the two linear equations will appear graphically. After making their predictions, students can plug their equation into their graphing calculators and sketch the function on the same graph as the linear equations. Observe what kinds of graphs the students have come up with. Invite a student up to graph the parabola on the Elmo. Ask students if this graph looks familiar (yes, from yesterday).  Go through the activity questions and emphasize the key connections between components of the lines to components of the parabola. Students will notice that the quadratic function has the same x-intercepts as the linear functions. The y intercepts will be a little bit trickier. If they are having trouble, as them to see how the two y intercepts of the linear functions relate to the quadratic function. Press for explanation from what different groups have noticed. They will notice that the y intercept of the quadratic function is the product of the linear y intercepts. Extend this discovery by asking them to focus on an x value and see if this relationship is still true (it is!). Check by focusing on one x intercept of a linear function and observing that because the y value is 0, then the y value for the quadratic will be 0 as well!  For questions 13 on, have students grab a ruler for each group and cover parts of the graph. This will emphasize that the product of the y coordinates of the linear functions can determine the sign of the y coordinate for any point on the graph. If the products are positive, the y coordinate will be positive and if the product is negative, the y coordinate will be negative. Have students fill out the sign chart and walk around to check for understanding. This resulting chart corresponds to a sign table that students traditionally used as an aid to graph functions.  After the chart is filled out, have them graph the quadratic function in problem 15 and walk around to observe how they are solving it. After a few minutes, prompt for different explanations on various components needed to find the quadratic function and sketch the parabola. Have them multiply the linear expressions algebraically, and find the resulting quadratic function. Give feedback on each student’s contributions, and congratulate them for constructing a parabola and finding a quadratic function from two linear functions! |
|  | **Assessments/Checks for Understanding:** Use various discursive moves to check for understanding and hold students accountable, mostly by inviting students to the board, pressing for explanation, and evaluating students contributions. At the end of the lesson, pose questions to the class that address student concerns (was the activity confusing, does it make sense, etc). After this activity, the class should discuss the material. Prompt students to share insight and ask questions. Homework will be handed out at the end of class and both the worksheet and homework will be handed in together to be graded. |
| 3 – 5 min. | **Closure/Wrap-Up/Review:**  Explain the homework for tonight involving working backwards from what they have done today (finding linear functions from a quadratic function). If there is time, work on the homework as an extension of the lesson and address any questions and concerns. Pose the question: “What if we multiply a quadratic and a linear function together?”  Have students fill out an exit ticket summarizing the main ideas they’ve taken away from today’s class, and once they hand it in, they receive their homework.  \*HW is similar to in class worksheet, but working backwards (starting with graph of a parabola and finding both linear equations) |
|  | **Self-Assessment:**  Did I make sure the students met the goals/objectives?  What were student concerns that I did not address/clarify?  What can I do during the start of tomorrow’s lesson that will easily connect lessons?  Were students engaged? Did students who participated feel acknowledged for their input, and did students who did not participate understand? How do I know they understand? |

**Building Quadratic Functions Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

1. What is the equation of the linear function shown to the right?



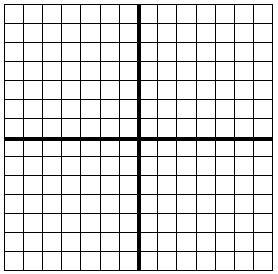
1. How did you find it?
2. The slope/*y*-intercept form of a linear function is *y* = *mx* + *b*. If you’ve written the equation in another form, rewrite your equation in slope/*y*-intercept form.
3. Now, factor out the slope, and rewrite the function as y = m(x + b/m). This is slope/x-intercept form.
4. Choose a second linear function and write it in slope/*y*-intercept form.
5. Graph the function on the axis above, and be sure to label it.
6. Rewrite your second function with the slope factored out (just like you did in Question 4).
7. For each function, what does (b/m) represent on the graph?
8. From their slope/*y*-intercept form, multiply the two functions together.
9. Graph the resulting function on the same axis as the two lines.
10. What kind of function did you get?
11. What relationship do you see between the graph from Question 10 and the lines?

- the *x*-intercepts?

* the *y*-intercepts?

1. Identify the left-most *x*-intercept on the graph. With a ruler, cover everything to the right of that point. What connections do you see relating the signs of the *y* -values?
2. Identify the right-most intercept on the graph. With a ruler, cover up everything to the left of that point. What connections do you see relating to the signs of the *y* -values?

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| --- | --- | --- |
| Y Value of L1 | Y Value of L2 | Parabola is Above/Below the X - Axis |
| + | + |  |
| + | - |  |
| - | + |  |
| - | - |  |



1. Based on the patterns you saw on the previous page, draw a sketch of the quadratic function that would be obtained from the linear expressions of these lines.
2. Write the equation for each line.
3. To check your sketch in Question 15, multiply the expressions together, and graph the resulting function on the grid above. How accurate was your sketch?

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| Day #: 3 | Lesson Title: Finding the roots and vertex of a quadratic equation |
| Goal: | To have students understand the process of finding roots in a quadratic equation, either visually or by using a graphing calculator. |
| Objectives: | * Students will know what a root of a function is, and how to find it with either a table, a graphing calculator, or to estimate is visually. * Understand how to solve any quadratic equation by knowing how to find roots of a function. For example, the equation 4x2 + 3x – 5 = 8 can be solved by finding the roots of f(x) = 4x2 + 3x – 13. * Students will understand the importance of finding roots (such as finding the time when a ball thrown into the air returns to the ground). |
| Lesson Summary (one paragraph maximum) | Students have already been introduced to quadratic equations, but now we will focus on the process of finding roots. We will first discuss the importance/necessity of finding roots of functions, and also the added benefits, such as being able to solve any quadratic equation. Afterwards, we will dive into the processes themselves. These will be estimating the roots visually (just looking at a graph), finding the roots by using a table of values (by hand or electronically), and we will also demonstrate how to find the “zeroes” by using a graphing calculator. |
| HW | Pg. 505; #3-5, 7a, 9  \*\**We are leaving out the material on finding the vertex for now, and will save that until the unit on maximization.* |

Shane Randa

Adam Poetzel

CI 402M

13 May 2010

Unit: Quadratic Models

Time Available: 50 minutes

**Day 4 Full Lesson Plan: *Factoring Polynomials***

**I. Goals**

* Students should be able to find roots of quadratic equations by factoring a given polynomial equation of non-factored form into factored form, which is **f(x) = a(x – r1)(x – r2)**, with **r1** and **r2** being the two roots.

**II. Objectives**

* Students should **recognize** the purpose for using the factored form, in that the roots of the function are found much easier in this form.
* Students should **identify** common factors prevalent in each term in a polynomial, whether integers or variables. They should also be able to **apply** the distributive property to “factor” out these common factors, which will simplify the process of converting a function to its factored form.
* **Compare** and **contrast** the general and factored forms of polynomials, and the purposes for each. For instance, how the general form exemplifies the degree, and the factored form exemplifies the x-intercepts. Students should also realize that both forms are equivalent.

**III. Prior Knowledge**

* Students should be well familiar with prime factoring of integers (knowing which integers a given number is divisible by). Being able to apply this will be essential to understand the idea of common factors in general.
* In the last few days, students have been introduced to quadratic functions, and the idea of solving for particular values, especially zero. They have seen how to find roots either visually or with a graphing calculator, and will now learn how to find roots in a third way.
* Students should already understand the distributive property, and be able to rewrite expressions via this property.
* Given two monomial factors of integer coefficients, such as (x – 3) and (2x+1), students can multiply these out (FOIL) to get a trinomial product.
* Students should be able to **define** a *root* of a function as a value for x (the input) that produces zero for y or f(x) (the output). Students should realize that a root is also an x-intercept of a function.
* Students already **understand** the zero product property and how it relates to finding roots of polynomials in factored form. They went over this 2 days ago.

**IV. Materials**

* Students should have notebooks and paper to take notes/ practice factoring problems individually.
* Some type of projection screen/area of focus, whether a SMART board, dry erase/chalk board, or an overhead projector.
* A Warm-up worksheet for each student, to be handed out as they enter class.
* An ELMO to use for going over the warm-up worksheet

**V. Motivation (~10 minutes)**

* As students walk in to the classroom, there will be a “warm-up” worksheet that each will receive. Once the bell rings, I will inform the students that they have 5 minutes to work on these problems, just to “get the mathematical blood flowing.”
* During this time, I will still make my usual rounds around the room, just to ensure that students are both attempting to complete the problems and doing so successfully. These warm-up problems cover material students should already be familiar with, but have topics (“FOILing”, zero product, common factors) that will relate to the lesson today.
* If, after several minutes passed, it appears that students are done with the worksheet, I will then proceed to bring the class’s attention front and center and go over the material as a class. (I am willing to allow a couple extra minutes for the class to work on the warm-up if there are any unanticipated struggles).
* When going over the worksheet, I will utilize the ELMO in order to ensure the correct answers to the warm-up problems are clearly seen (I won’t just call them out here). I will change up how I have each of the questions answered. I will likely have students represent each of the first two questions on the board or ELMO, since these are procedural. For the rest, I will likely just call upon an eager student. Since the students just reviewed zero product property and number 4, I anticipate several students to be able to answer these questions. Otherwise, I will have to scaffold the answer. “If three terms multiply to zero, what can we say about *at least* one of those terms?” or “what do we know about the product of 2 linear functions, especially relating to the case when one of the functions is 0?”
* For numbers 5 and 6, I anticipate students will be able to understand what a common factor is. If they have trouble with number 6, then I can start a brief discussion on the idea of factoring out “x” from all three terms. If any student has issue with this, I would ask “can anyone tell \_\_\_\_ why we can factor out an “x” in this case?” Hopefully from there the confused student will either understand that dividing 12x by x gets you 12, or I can stress the idea of x representing a number, and the same number is a factor of all 3 terms.
* After this, we come to number 7, which of course here the three terms have no common factors. I can then pose the question, “So if these terms have no common factors, would it then not be possible to factor the function f(x) = x2 **+** x – 6?”
* I’m anticipating some students to say no, since we cannot pull any common factors out of all three terms. Some students will likely say something along those lines too. I’m hoping that some student is savvy enough to realize that x2 + x -6 is the product of (x + 3) and (x – 2), since it was the answer to question 1 on the warmup. I’ll allow a discussion to go on for a couple minutes if no one brings this up, but once someone does I can say “so even though the three terms have no factors in common, we still have a way to factor their sum, based on the warmup.” If no student brings this up, then I’ll just bluntly ask “so no product of two terms can produce f(x)? What did we get for question 1 on the warmup?” after a couple minutes of discussion with no direction.
* I’ll then write f(x) = x2 + x – 6 = (x + 3) (x – 2) up on the board.

**VI. Lesson Procedure (20-35 minutes)**

* Once it is understood that the previous function can be factored into 2 binomials, I will introduce the lesson for the day. “So in that example, we changed f(x) = x2 **+** x -6 into f(x) = (x + 3) (x – 2). From our lesson 2 days ago, we know these are equivalent functions. But just as a review, what can we know immediately from looking at the second form (points to the right side of the function/equation on the board)?”
* I’m hoping that some students in the class realize that we can easily find the roots, since we have in a way factored a quadratic function into the product of two linear function. “That’s a great observation, \_\_\_\_\_! We can easily find the roots in this factored form, much easier than the general form? After all, it’s that wonderful zero product property coming into play again.”
* Just to check to make sure if other students are following along, I could also ask another student to justify the first answer. “So \_\_\_\_ says we can find the roots in this factored form much easier. Do you agree with this \_\_\_\_? What makes you think so?”
* If no one can respond with the fact that finding the roots becomes much easier in factored form, then I can pose the question “What about finding the x-intercepts, or the roots? If y (or f(x)) is 0, then what does the factored form tell us?”
* **Now I will define the factored form of a quadratic equation.**
* “So before we move on, I do want to officially define this for your notes, just to make sure we’re all on the same page.” (Place on board) **Factored form** (of a quadratic equation): f(x) = a(x – r1)(x – r2), with r1 and r2 being the two roots.
* “The “a” here represents any common factors that all 3 terms in the general form have in common. For example, if we had (on board) g(x) = 3x2 + 6x +3, we can factor out 3 and convert this to (on board) g(x) = 3(x2 + 2x + 1), and then the factored form of g(x) = 3(x+1)(x+1).”
* If a student asks how I went from the second to third step, I’ll tell them to just hold on to the suspense for a few minutes longer, and then I’ll show them how.
* “So if we only have one x2 term in the general form, like f(x) = x2 + x – 6, then the “a” term will always be one.”
* “So now we will discuss how we can factor a quadratic function in general form and just *convert* it into factored form. But just before I go over the procedure, I will give you a warning, and on the board, which means I also expect this in your notes. Ready?...ok!”
* (on board) **Not every quadratic function can be converted into factored form with integers.**
* “I only wanted you guys to put this in your notes, because I don’t want you to think this will be a once-and-for-all surefire thing. But when we can factor a quadratic, solving for the roots, and in some cases finding any value of y with a given x can be much easier to calculate.” “Why do you think it would be easier to calculate any y value this way?”
* If a student recalls Zane’s lesson discussing how having 2 linear binomials multiplied together to form a quadratic implies any y value of x for the quadratic is the two linear binomials’ y values multiplied together, then I can either reword what that student says, or even demonstrate an example just to reiterate.
* I could again take (x + 3) and (x – 2), and display both linear functions and their product on a TI calculator and project it in front on the class (I can even instruct the class to follow along). This is only review, but spending a couple minutes on a brief example can just ensure that students are better following along. I can show (like how Zane did) that for every value of x, the product of the linear equations’ y values is the same y value of the trinomial product.
* Now we will begin going over the procedure.
* “First off, just like in the warm-up, the first thing we want to do is find any factors that are common to all three terms of the trinomial.” (Put on the board: **Look for common factors**)
* I will also have an example for the class that I will use to better demonstrate the overall procedure. This example will be g(x) = 4x2 + 16x + 16. I can start off by saying “So based on this step, and the g(x), what can I do to this general form, if anything?”
* If no one responds, I can remind students to look for any factor common to all three terms, or in other words “given that not all three terms have x, does any number evenly divide the coefficients of all three of these terms?”
* I do, however, expect at least a few students to respond with “4,” and I could then have that student rewrite the function on the board below it (“great answer, would you mind showing us how the function would look after that?”). I will be sure to make sure that they don’t just rewrite “g(x) = x2 + 4x + 4” but instead “g(x) = 4(x2 + 4x + 4).” If that happened, I would just remind the student that the response without the “4” on the outside is not the same function (see EXTENSION 1).
* “Thanks \_\_\_\_. Now onto step 2. We know that the function must end in (x – r1)(x – r2). Now we just have to find those two roots. We can start off by writing down 4(x + \_\_\_\_) (x + \_\_\_\_), we will have to fill in the blanks. It might also be to your benefit to do the same in your notebooks.”
* “So for the x2 + 4x + 4 part we are factoring, we need to find 2 numbers to fill in these blanks, and here’s how you do it.” (on board) **for f(x) = x2 + bx + c,** **the two terms we fill in the blanks must add to b, while multiplying to c.**
* “If you remember from FOILing out two binomials, such as (x + n)(x + m), we would get x2 + mx + nx + nm as a product (we’ve already discussed this in the warm-up), which is also x2 + (m+n)x + nm. Now for factoring, we are only (on board) “FOIL-ing in reverse!””
* “So I want two numbers that add to 4 and multiply to 4, since b = 4 and c = 4. How can this happen?” I will allow several seconds for wait time, and either a student will realize that 2 can work for both blanks, or I can scaffold the class to this answer….
* …”So what integers multiply to 4? And just a heads up, we can use both negative and positive integers?” From here, I can draw a list on the board of possible combinations (this will especially work for other, more complicated examples, which is why demonstrating this now is so crucial). For each one, such as 1 and 4, -1 and -4, 2 and 2, etc., we can check the sum, and realize that only 2 and 2 will suffice.
* “Now we are left with g(x) = 4(x+2)(x+2), or g(x) = 4(x+2)2 (put both on the board). So based on this, what would my roots be?” Wait for someone to respond. If no one does, I can then ask “so for values of x will g(x) be zero?” to generate their thought process.
* Of course, here the answer is x = -2 only, which I can then define on the board as a **double root.**

**VII. Assessment**

* The warm-up in the beginning of the class will help us to not only get the class thinking about factors before the lesson, but will also help us to ensure the class is grasping the concepts from the last couple days (zero-product and roots for example). After all, these concepts are essential to make this lesson easily comprehensible.
* Throughout the lesson, I am periodically inviting student participation to help me out with the factoring examples. Students will figure out common factors and even demonstrate thinking on the board. There are several opportunities for me to make sure the class is following along.
* The homework assigned will help me to assess student learning both today and tomorrow. I am planning on giving students around 10 minutes to work on the homework. The first few problems are relatively simple, and very similar to what was done as a class. I should hopefully be able to catch any misconceptions students have before the bell rings, and clear anything else up once the homework is finished.

**VIII. Closure**

* “So as you can see, once we can factor a quadratic function into the factored form, we can find the roots, and really any value of x, much, much faster and easier in most cases. This can help with both quadratic functions, and even just quadratic equations, no matter what the value on the other side is.”
* Depending on the time remaining, I can go through more examples (such as f(x) = x2 -2 -15, or h(x) = 3x2 + 24x + 36) in the same way, just to try and cover as broad an array of possible cases as possible.
* “You guys now have the rest of the time to work on your homework for the day (if time is left at all, but if there’s sufficient time left, we can go through some extensions). As always, feel free to work on the homework with one or two of your peers, but just remember that your still responsible for your own work ☺.”

**IX. Extensions**

1. I could discuss the differences between the two functions (one being 4 times the other). For example, are the x-intercepts the same? The y-intercepts? If not, how do they differ? Once students realize that these two functions do indeed have the same roots, then they can realize that any common non-variable factor of a quadratic function being solved for 0 can just be “cancelled out.”
2. What about when the coefficient of the x2 term is not 1, and we can’t factor that coefficient out? I can spend any extra time discussing how we factor these cases. After all, we can’t just place it into an (x – r1)(x – r2) form. So I could provide examples where the factored form is (3x + 3)(x – 2) for example, which is of course done by a more detailed form of guess and check, based off an adjustment of what the possible roots can be.

**X. Standards**

* **6.A.4:** Identify and apply the associative, commutative, distributive, and identity properties of real numbers, including special numbers such as pi or square roots.
* **6.B.3b:** Apply primes, factors, divisors, multiples, common factors, and common multiples in solving problems.
* **8.A.3a:** Apply the basic properties of commutative, associative, distributive, transitive, inverse, identity, zero, equality, and order of operations to solve problems.
* **8.D.4:**  Formulate and solve linear and quadratic equations and linear inequalities algebraically and investigate nonlinear inequalities using graphs, tables, calculators and computers.

**May 8th, 2010 Warm-up**

1. Find the product of (x + 12) and (x - 2)
2. Find the product of (x + 3) and (x – 2)
3. Find all possible values for x: **x(x+5)(x-3) = 0**
4. Let f(x) = x + 5, g(x) = x-1, and h(x) = g(x)\*f(x)

Write out the function h(x) in quadratic general form, and give it’s x intercept(s) and y intercept.

1. Of the terms 27 and 39, are there any factors common to both?

If so, what are they?

1. Of the terms 24x2, 12x, 18x, are there any common factors of all three?

If so, what are they?

7. Of the terms x2, x, and -6, are there any common factors of all three?

If so, what are they?

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| Day #: 5 | Lesson Title: Quiz |
| Goal: | To give a quiz on solving quadratic equations using linear functions, finding the roots, and factoring quadratic equations. |
| Objectives: | For students to solve quadratic equations by breaking them down into linear functions and vice versa, factor quadratic equations, and find the roots of a quadratic equation by letting y equal 0 and solving for x. |
| Lesson Summary (one paragraph maximum) | Since the day is 25 minutes, this will be the perfect time for short quiz. This quiz will only be on the past three days because it is important for the teacher to assess student’s understanding early on in the unit. If students are not where we think they should be, we will know whether or not we should move on to vertex form or if we should spend extra time on the material we have covered. |
| HW | No HW. |

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| Day #: 6 | Lesson Title: Vertex Form |
| Goal: | The goal of the lesson is to have students use vertex form to graph equations and switch between vertex and general form. |
| Objectives: | Students will learn how to translate a quadratic function x^2 by using vertex form.  Students will change a quadratic equation from vertex form into polynomial form and vice versa |
| Lesson Summary (one paragraph maximum) | We will briefly go over last week’s quiz and review. Then we will show how to translate quadratic formulas starting from y = x^2. From this, students will learn vertex form. We will graph various equations in vertex form by looking at the translations. Then, we will go from vertex form to general form, and learn how to do the opposite. |
| HW | 2-10 (evens) |

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| Day #: 7 | Lesson Title: GeoGebra (\*technology\*) |
| Goal: | To get students to explore manipulating quadratic formulas using the vertex and general forms of the equation. |
| Objectives: | Students will be able to identify the vertex and translations from animation on Geogebra.  Students will be able to describe movement as a variable changes in the equation.  Students will compare both forms of the equation and how the graphs change as variables are manipulated |
| Lesson Summary (one paragraph maximum) | In the computer lab, students will be set up with an application in GeoGebra that already has vertex and general form defined. With some instructions, they will be able to see their variables and how they change as the parabola’s are animated. Students will have a short worksheet describing what they see as different variables change. |
| HW | NO HW |

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| Day #: | Lesson Title: Quadratic Formula |
| Goal: | For the students to have an understanding of the quadratic formula and that it can be used for any quadratic equation that is in general form. |
| Objectives: | The students will know how to put a quadratic in general form and be able to use the formula. The students will memorize the quadratic formula because it is a means of solving a quadratic when it cannot be factored. |
| Lesson Summary (one paragraph maximum) | The focus of this lesson is going to be getting the students to understand that the quadratic formula is a way to solve any quadratic. It may not always be the easiest way because factoring might be a better alternative. The basis of the lesson is going to be around a fun way of memorizing the formula: “A negative boy couldn’t decide whether or not to attend a radical party. He was square so he missed out on 4 awesome chicks. The party was over at 2 a.m.” This is a way that students can memorize the formula in a way that is not just a cluster of letters and mathematical signs. With this group of students, this is an important aspect of our teaching. |
| HW | P. 568-569 #1-7, 10, 12a |

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| Day #: 9 | Lesson Title: Quadratic Formula (continued) and Review |
| Goal: | We will wrap up our lesson on the Quadratic Formula and review topics discussed for the past two weeks to prepare for the end of the week quiz tomorrow. |
| Objectives: | Students will learn how to use the quadratic formula when given a quadratic equation.  Students will learn which method is best for solving quadratic equations. |
| Lesson Summary (one paragraph maximum) | We will do a review of what we learned with the quadratic function yesterday and compare and contrast all four ways of solving quadratic functions and what they are best used for. We will explore what happens when b^2 – 4ac is 0 or when it is negative. We will practice substituting numbers in for a, b, and c and using the calculator to check our answers. I will hand out a review sheet and give the students time in class to work on it and address any questions or concerns. |
| HW | Review Sheet |

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| Day #: 10 | Lesson Title: Quiz and Introduction to Maximization \*problem based\* |
| Goal: | Assess students understanding of Vertex Form and General form, as well as the Quadratic Formula. Students will then be introduced to a problem involving maximization. |
| Objectives: | Students will illustrate their learning of vertex form, general form, and the quadratic formula on the quiz.  Students will be exposed to a problem with maximization using quadratics and predict how to go about solving the problem. |
| Lesson Summary (one paragraph maximum) | At the beginning of class, students will be given a quiz to assess their understanding of the material from the past week. After the quiz, students will be introduced to a problem that involves maximization using quadratic functions. The problem will ask how area can be maximized with a given perimeter. Students will discuss possible ideas about how to go about solving the problem and test different claims. |
| HW | NO HW |

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| Day #: 11 | Lesson Title: Maximization (continued) \*problem based\* |
| Goal: | Students will work in groups and solve challenging, practical maximization problems and reflect on their experiences. |
| Objectives: | Students will pose a prediction on how to solve the problem.  Students will work in groups and collaborate to find the largest area of a yard, as well as the highest point of a tossed object.  Students will graph the function and find the vertex of the parabola. |
| Lesson Summary (one paragraph maximum) | Students will be posed the same problem from last Friday and work in groups to solve the problem, as well as a problem finding the highest point of an object in projectile motion. Students will reconvene as a class and work on the problem together. An extension will be a minimization of costs problem, which will be worked on in class and for homework.. |
| HW | Worksheet on minimization. |

Kevin Mauser

Final Lesson Plan

CI 402

Course: Algebra 1 (50 Minutes)

Topic: Cubic Functions

1. Goal
   1. To develop an understanding of cubic functions, seeing the differences and similarities to quadratics.
2. Objectives
   1. The students will be able to find the roots of a cubic function
   2. Students will be able to graph a cubic function
   3. Students will be able to come up with an equation of a cubic function given its graph
   4. Students will be able to take a factored out cubic function and put it in general form
   5. Students will know how to take a cube root, and if a number is a perfect cube
3. Prior Knowledge
   1. Students will know how to find roots of a quadratic equation
   2. Students will be familiar with factoring polynomials
   3. Students will know the difference between the various forms polynomials can be written in
   4. Students will know how to find the volume of a prism
4. Materials
   1. Graphing calculators
   2. SMART Board
   3. 4 unit cube for demonstrating volume
   4. Dry Erase board for each student
5. Lesson Procedure
   1. “Alright class, today we are going to use our knowledge of quadratic functions and explore a new topic: cubic functions.” I will then draw a 4 unit by 4 unit square on the board and ask, “First off, what do we know about the area of a square with sides of 4 units long?” From here I expect students to say something about length times width or side squared, giving us 16 units2, which is correct. From here I can demonstrate that area of a square = side2 which models f(x) = x2. This is where the relationship between area and volume come into play.
   2. This is when I will take out the 4 unit cube and ask the students, “What is the volume of this cube, which has a side length of 4 units?” Students should remember previously that volume of a prism is length times width times height or in the case of a cube, that it is side cubed. Therefore, the correct answer is 64 units3. In this case of a cube, I can then write that we got this answer by the formula: volume of a cube = (edge length)3. Just like the area of a square modeled f(x) = x2, the volume of a cube models f(x) = x3.
   3. I will then write down what we have just done on the board, that 43=64 so the students can see with their calculators that this is correct. It seems that in my experiences, students feel reassured when they can check the answers on the calculator.
   4. This is when the definition of a perfect cube will come into place, as 64 is a perfect cube. “I would like to introduce everyone to a term that you should put in your notes, and that is a perfect cube. Just as we have learned that a number whose square root is an integer is a perfect square, a number whose cube root is an integer is said to be a perfect cube.” I think it is important to continue to compare cubics to quadratics because it eases the students into a more difficult topic.
   5. “At this time I would like everyone to take out their dry erase boards because we are going to look at a few examples and I would like for you to tell me if they are perfect cubes or not.” At this point I will put several examples of perfect cubes and non-examples of perfect cubes on the SMART Board and walk around to assess how the students are doing with perfect cubes. This allows me to have some students come up to the board and show why or why not the numbers are perfect cubes without making them feel embarrassed in front of their classmates if they have the wrong answer.
   6. This is the time that I would like to have the students start to look at graphs of cubic functions. “Can everyone please take out your graphing calculators and graph the function f(x) = x3.” It is important that the students know what the most basic of cubic functions look like, and drawing one by hand is not the most accurate and precise way of demonstrating this.
   7. I would like for them to understand that the positive x-values of the x3 is the similar to that of x2, but the negative x-values are going in a different direction because of the negative signs not cancelling out.
   8. At this time I would like to look at transformations of the basic cubic function, f(x) = x3. “Just as we did with quadratic transformations, cubic functions can be transformed in the same way from f(x) = x3. Would everyone please tell me what the transformation of f(x) = (x+2)3-4 is.” I will use wait time here to give the students time to figure out that it is left two units and down four units. This will also give me a chance to make my way around the room, and answer individual questions if there are any. In a class that has 50% underachieving students, I anticipate questions to arise and often the questions will not be asked to the whole class because some of the students “feel stupid” or inferior. Consequently, I feel as though it is important to create a classroom that is welcoming to questions and wrong answers because typically if one student has a question there is going to be someone else in the class who has that same question. It is important to bring this up and praise the student for asking that question and answer it to the class as a self-esteem booster as well as a way of benefitting the entire class.
   9. After going over a few more examples of transformations, it will be time to move on to finding the x-intercepts of cubic functions. I would first like to look at the graph of x3 to show that, although cubic functions can have up to three roots, they do not necessarily have that many, as this graph only has one, at x=0. To show an example of a cubic function with more than one root, I will give the example y = (x+3)(x+1)(x-2) to the class and ask them to tell me what the roots are. This example will hopefully not take too long, as I have given the students the factored form, and they hopefully will remember from the previous sections that in order to find the roots we set each of these factored parts equal to zero and solve to find a root. However, I anticipate that it will take a little longer than I hope because this is a difficult concept for students to master.
   10. After looking at a few more examples similar to this, I would like to explore the topic of finding the equation when we are given the graph of a cubic function. The example that I am going to use has three x-intercepts: x=0, -1, and 2. We are also given the point (1,4). As x goes from negative infinity where the y value is very positive, the y-values grow increasingly negative as x goes towards positive infinity. Thus, we know that it will have a negative sign out front to show the reflection over the x-axis. I will have this graph posted on the SMART Board for the students to see.
       1. I will use this time to walk around and see how the students are doing, reminding them what we know about roots of functions, scaffolding them to think about the factored form of functions.
       2. I am anticipating that several students will give the equation to be y = x(x+1)(x-2) without taking into consideration the point that is given (1,4) or the reflection over the x-axis. Therefore, I will stress to the students to check their answers by saying, “Before you raise your dry erase boards, make sure and ask yourself, ‘Does this graph match my equation at all points?’” I want the students to think about whether or not their answer makes sense, rather than just getting it done quickly because it is more important to get the correct answer than to be the first person done with the wrong answer.
   11. At this time, I would like to go over one more example similar to this one allowing students to learn from the mistakes they made in the first example.
6. Closure
   1. Every class needs to be brought back together at the end, as a review of what was discussed that day. I feel as though math journals can be a useful tool, although I have never used them in my experiences as a student or a teacher. “At this time, I would like everyone to take out their math journals and write down two things that we have learned today.” While the students are writing down their thoughts, I would like to say, “Now class, as you can see from what we have gone over today, cubic functions are extremely similar to what you already know: quadratics. When we are given the equation in factored form, we can easily find the roots. With the right points we can graph any cubic function, and we also looked at perfect cubes and how similar they are to perfect squares.” I feel as though this much material may be overwhelming if it is not stressed to the students that it really is not that much more material, just an extension of quadratics.
7. Extensions (Time Permitting)
   1. There will be homework for the students from the book, page 575 #1-5, 7, 8, 11. Therefore, if there is time students will be given time to begin their homework allowing time to walk around answering questions that they may have. I feel as though there will not be a whole lot of extra time with this topic because a lot is discussed with cubic functions.
8. Assessment
   1. I will get an understanding of where the students are at with this topic the following day when they turn in homework.
   2. Also, the dry erase boards allow me to walk around during class and see how everyone is doing.
   3. With my approach to this lesson by expanding upon quadratics, how quickly everyone is grasping the material will also allow me to get a gauge on how well I taught quadratics.
9. Standards
   1. Number Sense
      1. With this lesson, we are exploring perfect cubes and cube roots, and what these numbers represent.
   2. Algebra and Analytic Methods
      1. We are using factored forms and showing how they can be manipulated algebraically and put in different polynomial forms. We are also using graphing technology to compare and predict cubic functions with that of quadratic functions
   3. Geometry
      1. We are exploring how the area of a square is a quadratic function and the volume of a cube is a formula that can be represented as a cubic function
   4. Problem Solving
      1. Allowing students to work individually and think critically
   5. Communication
      1. Working with the dry erase boards to promote classroom discussion and to survey the class
      2. Requiring students to come to the board to explain the answers; how you get an answer is just as important as the answer
   6. Making Connections
      1. We compared the area of a square to a quadratic function and the volume of a cube to a cubic function
   7. Using Technology
      1. The SMART Board is an extremely useful tool that students love to come up and show their work. The dry erase boards are simple, but get the job done and promotes a game-like atmosphere in the classroom

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| Day #: 13 | Lesson Title: Review for Chapter Test |
| Goal: | To play a jeopardy style review game to prepare the students for the chapter test that is being given the following day. |
| Objectives: | To break the class up into teams to promote good teamwork skills. We are also looking to get the students prepared for the test so that they have sample problems to look over and see what material they need to study more exclusively. |
| Lesson Summary (one paragraph maximum) | We will design a jeopardy game to get the students engaged in a fun activity to sum up what we have covered in this chapter. The class will be divided into three teams. Each member of the team will go around and choose a question, and that team has the opportunity to answer the question first, but if they are incorrect, another team can steal the points. We feel as though this promotes a sense of getting the correct answer instead of simply being the first person finished with the problem. |
| HW | P. 545-546 #1-14 but not #7 |

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| Day #: 14 | Lesson Title: Chapter Test |
| Goal: | To formally assess that the students have an understanding of quadratic models with a full period test. |
| Objectives: | We want to see if the students can solve equations using a calculator’s tables and graphs, by factoring, and by applying the quadratic formula. |
| Lesson Summary (one paragraph maximum) | There are several types of questions we plan on including on this test. We want the students to be able to solve quadratic equations in a number of ways: using a calculator, factoring, and the quadratic formula. Due to the use of a calculator, we were planning on having a two-part test, one that students can use a calculator and another part where the use of a calculator is not permitted. This will allow us to assess whether or not our students understand the material or are simply relying on technology to do it for them. |
| HW | Have a good weekend. |

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| # | Learning Objective | Assessment Item |
| 1 | The students should be able to recognize how a quadratic equation varies from y=x2 | Given: y = -3(x-4)2 +2  Describe the transformations on the graph of y = x2 that give this parabola.  Solution:  Reflected across x-axis, vertical stretch with factor of 3, and translation right 4 units and up 2 units. |
| 2 | Students should know how to use the quadratic formula. | Solve y = 2x2-5x+3 using the quadratic formula.  Solution:  a=2  b=-5  c=3  discriminant = 1  x = 3/2, 1 |
| 3 | Given a cubic function in factored form, students must be able to graph the function without a calculator. | Given: y = -(x+2)(x-3)(x+1)  Solution:  x-intercepts: x = -2,-1,3  y-intercepts: y = 6 |
| 4 | Given a trinomial function in general form, students should be able to convert it into factored form and thus find the roots. | *Find the root(s) of f(x) = 2x2 + 10x – 48 by converting f(x) into factored form.*  **Solution:** f(x) = 2(x2 + 5x – 24)  f(x) = 2(x + 8)(x – 3)  Set f(x) = 2(x +8)(x – 3) = 0  f(x) = 0 if x = -8 or x = 3, so the roots are at x = -8 and x = 3 |
| 5 | Students should know how to find the linear equations that make up a given quadratic function (or given parabola on the graph) | Find the linear functions of the parabola shown.  Solution:  x-intercepts: x = -2, 2  y-intercept: product of y intercept = 4  left of x = -2 and right of x = 2 is negative.  Linear functions are f(x) = x-2 and f(x) = x+2 |

